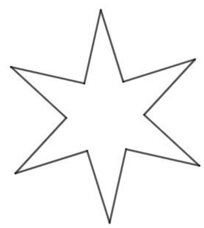
Continuous Round

October 24th, 2020

1. **[2]** Below is a 6-pointed star whose sides are all 1, and the angle at each point is 30 degrees. The points are evenly spaced. If the area of the star can be expressed as $a\sqrt{b}-c$ with a, b, c all integers and b not divisible by the square of any positive integer, find $a+b+c$.



2. **[2]** Given that $2020x^{2019}+x^{2018}+x^{2017}+\ldots+x^2+x-2019$ has exactly one real root that can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers, find $m+n$.

3. **[3]** If the distance between the two intersections of $y=(2-\sqrt{3})x$ and $x^2-20x+y^2=0$ can be written as $a\sqrt{b}+c\sqrt{d}$ where b and d are not divisible by the square of any prime, find $a+b+c+d$.

4. **[3]** In rectangle $ABCD$, $AB=15$ and $BC=5$. Pentagon $WRATH$ is inscribed in $ABCD$ such that TW is parallel to BC and HR is parallel to AB. Suppose that $\angle THW=135^\circ$ and $\angle HWR=90^\circ$. If HW^2 can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers, compute $m+n$. (Note every vertex of $WRATH$ lies on a side of rectangle $ABCD$)

5. **[5]** Let $H(x)$ denote the inverse of the function $x2^x$ defined for all positive reals. Compute the remainder when
$$\sum_{k=2}^{2048}\lfloor H(k)\rfloor 2^{\lfloor H(k)\rfloor}$$
is divided by 2048.

6. **[5]** Let ABC be a triangle with $AB=8$, $BC=7$ and $CA=5$. Let ω denote the circumcircle of ABC. The tangent to the ω at B meet AC and the tangent at C to ω at points L and D, respectively. Let the circumcircle of ABD meet segment CL at K. Given that the length of DK can be expressed as m/n for relatively prime positive integers m and n, what is $m+n$?

7. **[6]** Billy was given a polynomial $p(x)$ by his teacher. His teacher instructed him to write $p(x-1)$ in standard form (i.e. of the form $a_nx^n+a_{n-1}x^{n-1}+\cdots+a_0$). Miraculously, the coefficients of $p(x)$ and $p(x-1)$ were the same except for one term! Given that $p(0)=0$ and $p(1)=1$, find the sum of all possible values of $p(5)$ between 0 and 1000 inclusive.

8. **[7]** Let $ABCD$ be a parallelogram. Let M and N be the midpoints of CD and BC, respectively. Suppose the circumcircle of $\triangle MAN$ meets AB at X. If $AB=150$, $AD=90$, and $\angle DAB=120^\circ$, find AX.

9. **[8]** Nyle is playing around with the sequence $a_n = 34a_{n-1} - a_{n-2}$ for $n \geq 3$. He finds that every single term in the sequence is 98 more than a perfect square! Given that $a_1 = 102$, find a_2.

10. **[9]** Let ω be the circumcircle of $\triangle ABC$, and let a point D distinct from the vertices be on ω. Let M and N be the midpoints of AB and AC, and let DM meet the tangent to ω at B at E, DN meet the tangent to ω at C at F, and DA meet BC at G. Suppose $EF||BC$ and $FG||BE$. Given that $AB = 20$ and $AC = 12$, find the perimeter of $ABDC$.