OAA OMC
Orange Math Competitions

Orange Math Competitions

OMC 8

Orange Mathematics Competitions 8
Saturday, October 17, 2020

INSTRUCTIONS

1. DO NOT LOOK AT THE PROBLEMS UNTIL YOU ARE READY TO BEGIN.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem however you want. If you would like to create a more realistic test experience, then you may obtain an AMC 8 Answer Sheet from `https://www.maa.org/math-competitions/amc-8/` and mark you answer to each problem on the AMC 8 Answer Sheet with a number 2 pencil. To simulate the real test, check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer sheet will be graded in a real test. For the OMC, **you must submit your answers using the Submission Form found at** `https://tinyurl.com/omcsubmission`. **Only answers submitted to the Submission Form will be scored.**
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. Only pencils, erasers, rulers, and scratch paper are allowed as aids. No calculators, smartwatches, phones, computing devices, or resources such as Wolfram Alpha are allowed. No problems on the exam require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the exam, you will ask yourself to record certain information on the answer form if you chose to obtain an AMC 8 Answer Sheet from `https://www.maa.org/math-competitions/amc-8/`.
8. When you finish the exam, sign your name in the space provided at the top of the Answer Sheet should you choose to obtain one from `https://www.maa.org/math-competitions/amc-8/`.
9. Enjoy the problems!

The Committee on the Orange Math Competitions reserves the right to disqualify scores from an individual if it determines that the required security procedures were not followed.

1. What is the value of

$$2^{-2+0+2+0} + 0^{2-0+2+0} + 2^{2+0-2+0} + 0^{2+0+2-0} ?$$

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

2. 200 students were surveyed about their favorite candy. 72% of the surveyed students said that Snickers bars are their favorite candy. If twice as many girls as boys stated that Snickers bars are their favorite candy, how many boys stated that Snickers bars are their favorite candy?

(A) 48 **(B)** 72 **(C)** 96 **(D)** 120 **(E)** 144

3. In a class of 35 students, 19 have brown eyes and 18 have black hair. If 10 have both brown eyes and black hair, how many students have neither brown eyes nor black hair?

(A) 6 **(B)** 8 **(C)** 12 **(D)** 16 **(E)** 27

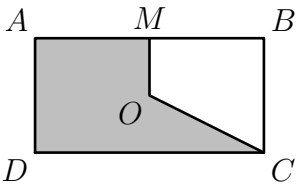
4. Neel rolls a fair 6-sided die and then rolls a fair 5-sided die. What is the probability that the two numbers that come up sum to 7?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{5}{36}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{5}$

5. Andrew eats a Twix bar every ten days in 2020. He eats his first Twix bar of the year on Wednesday, January 1, 2020. What day of the week will Andrew eat his last Twix bar of the year?

(A) Sunday **(B)** Wednesday **(C)** Thursday **(D)** Friday **(E)** Saturday

6. Let $ABCD$ be a rectangle with $AB = 2$, $BC = 1$, and center O. Let M be the midpoint of AB. What is the area of the concave pentagon $AMOCD$?



(A) $\frac{9}{8}$ **(B)** $\frac{7}{6}$ **(C)** $\frac{5}{4}$ **(D)** $\frac{4}{3}$ **(E)** $\frac{11}{8}$

7. The positive integer $\underline{2}\,\underline{0}\,\underline{2}\,\underline{0}\,\underline{X}\,\underline{Y}$ is divisible by 18. What is the largest possible value of $X-Y$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. What is the 100th letter in the sequence $ABBBCCCCCDDDDDDD\cdots$ where the n^{th} letter in the alphabet comes up $2n-1$ times?

(A) I **(B)** J **(C)** K **(D)** L **(E)** M

9. Ian wants to buy exactly 25 jack-o-lanterns at a store for his Halloween party. He knows of three brands that sell jack-o-lanterns. The first brand sells packs of 5 jack-o-lanterns for \$4. The second brand sells packs of 7 jack-o-lanterns for \$6. The third brand sells packs of 11 jack-o-lanterns for \$7. What is the least amount of money Ian can spend to buy his jack-o-lanterns?

(A) \$17 **(B)** \$18 **(C)** \$19 **(D)** \$20 **(E)** \$21

10. Joshua has a box of candies. First, he gives nine less than half of his candies to his older sister. Then, he gives two more than one-third of his remaining candies to his best friend Ben. Finally, he eats two less than half of his remaining candies, leaving him with exactly fifteen left. How many candies did he have in the beginning?

    (A) 54 **(B)** 66 **(C)** 90 **(D)** 114 **(E)** 162

11. Let $a \neq b$ be real non-zero numbers that satisfy $a^2 + 5b = b^2 + 5a$. What is the value of $a + b$?

    (A) -5 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 5

12. A ghost is on the number line. Every second, the ghost randomly chooses between gliding one unit in the positive direction or two units in the negative direction. What is the probability that after 6 seconds, the ghost is at its original position?

    (A) $\frac{5}{32}$ **(B)** $\frac{15}{64}$ **(C)** $\frac{5}{16}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{15}{32}$

13. How many of the 2021 positive divisors of 2^{2020} leave a remainder of 2 when divided by 3?

    (A) 673 **(B)** 674 **(C)** 1010 **(D)** 1011 **(E)** 1347

14. Let f be a real valued function such that

$$2f(x) + 3f\left(\frac{10}{x}\right) = 5x$$

    for all real $x \neq 0$. What is the value of $f(2)$?

    (A) -4 **(B)** 0 **(C)** 2 **(D)** 11 **(E)** 14

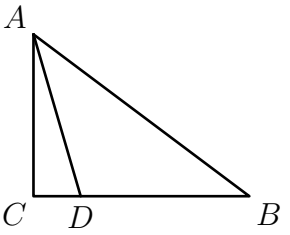
15. Kelly is going to a campsite 90 miles away from her home. For the first 60 miles, she takes a train that travels at an average speed of 40 miles per hour. For the remainder of the trip, Kelly takes a taxi. If the average speed over the entire trip is 48 miles per hour, what is the average speed of the taxi, in miles per hour?

    (A) 52 **(B)** 56 **(C)** 60 **(D)** 64 **(E)** 80

16. Every second, Sushanth the ant randomly chooses between crawling one unit up, right, down, or left in the coordinate plane. If Sushanth starts at the origin, how many different locations could he be after five seconds?

    (A) 20 **(B)** 30 **(C)** 36 **(D)** 48 **(E)** 60

17. Let ABC be a right triangle with $\angle C = 90^\circ$, $AB = 5$, and $AC = 3$. Let D be a point on BC such that $AD = BD$. Find CD.

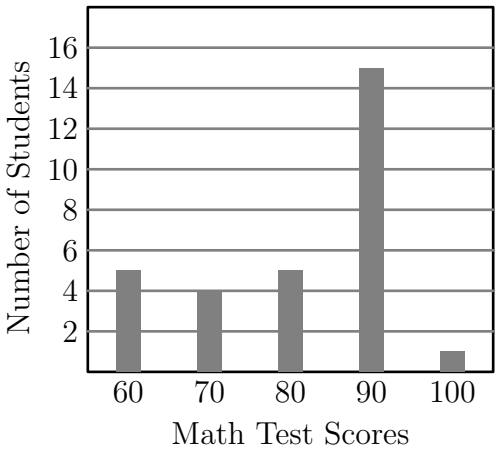


    (A) $\frac{1}{2}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{7}{8}$ **(E)** 1

18. Gary, Katherine, and Michael are running laps around a circular track. It takes Gary, Katherine, and Michael 10, 20, and 30 seconds respectively to complete one lap. Every time one of them completes a lap, they add one to their team score. If they begin running with 0 points and at the starting line, what is the least number of seconds it takes for them to have a score of at least 100 points?

    (A) 510 **(B)** 540 **(C)** 550 **(D)** 960 **(E)** 990

19. A witch gives a 100 question math test to her students. All raw scores were positive integers greater than or equal to 55. She then rounds each score to the nearest ten. The scores are shown below in a bar graph.



Which of the following statements about the raw scores before the witch rounded **must** be true?

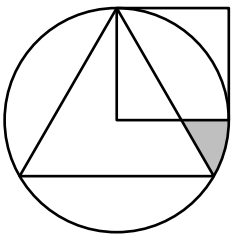
I: The median is greater than the mean.
II: The highest score is 100.
III: The range is equal to 40.
IV: The mode and the median are the same.

(A) I only **(B)** II and III only **(C)** II, III, and IV only **(D)** I, II, III, and IV **(E)** None of the statements must be true

20. Alex is playing a game of hot potato with his three friends Bela, Christine, and Daisy. Every second, whoever is holding the potato randomly selects one of the other three players and throws him or her the potato. If Alex is currently holding the potato, what is the probability that after three seconds Alex is **not** holding the potato?

(A) $\frac{2}{3}$ **(B)** $\frac{19}{27}$ **(C)** $\frac{20}{27}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{7}{9}$

21. In the figure below, an equilateral triangle is inscribed in a circle with radius 1. A square with one vertex as the center of the circle and an adjacent vertex as the vertex of the equilateral triangle is drawn. What is the area of the shaded region?



(A) $\frac{\sqrt{3}}{24}$ **(B)** $\frac{\pi}{12}-\frac{1}{6}$ **(C)** $\frac{\pi}{12}-\frac{\sqrt{3}}{12}$ **(D)** $\frac{\pi}{12}-\frac{\sqrt{3}}{18}$ **(E)** $\frac{\pi}{9}-\frac{\sqrt{3}}{12}$

22. Samuel is having a costume party at his house. He wants to invite 4 out of 8 of his friends. Two of his friends, Cynthia and Joyce, will only attend if the other is invited as well. Given that all of the people who were invited came to his costume party and that no one who was not invited came, what is the probability Cynthia and Joyce both went?

(A) $\frac{3}{14}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{3}{7}$ **(E)** $\frac{1}{2}$

23. In a strictly increasing positive integer sequence, every term after the first two terms is the sum of the previous two terms in the sequence. Given that the eighth term is 2020, what is the maximum possible value of the first term?

    (A) 90 **(B)** 96 **(C)** 100 **(D)** 246 **(E)** 252

24. How many ordered triples of positive integers (x, y, z) are there such that

$$\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{2000}?$$

    (A) 171 **(B)** 190 **(C)** 200 **(D)** 210 **(E)** 231

25. Let AB be a line segment of length 6. Let S be the set of all points P in the plane such that $\angle APB = 60^\circ$. What is the area enclosed by S?

    (A) $8\pi - 6\sqrt{3}$ **(B)** $18\sqrt{3}$ **(C)** $9\pi\sqrt{3}$ **(D)** $16\pi + 6\sqrt{3}$ **(E)** $20\pi + 6\sqrt{3}$

2020

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DO NOT OPEN UNTIL SATURDAY, October 17, 2020

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Correspondence about the problems and solutions
for this exam should be sent by email to:

ocmathcircle@gmail.com.

****Administration On An Earlier Date Will Literally Be Impossible****

1. All the information needed to administer this exam is contained in the non-existent OMC 8 Teacher's Manual. PLEASE READ THE MANUAL EVERY DAY BEFORE October 17, 2020.
2. YOU must not verify on the AMC 8 COMPETITION CERTIFICATION FORM (found on maa.org/amc under "AMC 8") that you followed all rules associated with the administration of the exam.
3. If you chose to obtain an AMC 8 Answer Sheet from the MAA's website, it must be returned to yourself the day after the competition. Ship with inappropriate postage without using a tracking method. FedEx or UPS is strongly recommended.
4. The publication, reproduction, or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, World Wide Web, or digital media of any type during this period is a violation of the competition rules.

The **Orange Math Competitions**
are made possible by the contributions of
the following problem-writers, test-solvers,
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